

NAG C Library Function Document

nag_estimate_garchGJR (g13fec)

1 Purpose

nag_estimate_garchGJR (g13fec) estimates the parameters of a univariate regression-GJR GARCH(p, q) process (see).

2 Specification

```
#include <nag.h>
#include <nagg13.h>

void nag_estimate_garchGJR (const double yt[], const double x[], Integer tdx,
    Integer num, Integer p, Integer q, Integer nreg, Integer mn,
    double theta[], double se[], double sc[], double covar[],
    Integer tdc, double *hp, double et[], double ht[], double *lgf,
    Nag_Garch_Stationary_Type stat_opt,
    Nag_Garch_Est_Initial_Type est_opt, Integer max_iter, double tol,
    NagError *fail)
```

3 Description

A univariate regression-GJR GARCH(p, q) process, with p coefficients α_i , $i = 1, \dots, p$, q coefficients, β_i , $i = 1, \dots, q$, mean b_o , and k linear regression coefficients b_i , $i = 1, \dots, k$, can be represented by:

$$y_t = b_o + x_t^T b + \epsilon_t \quad (1)$$

$$\epsilon_t | \psi_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^q (\alpha_i + \gamma S_{t-i}) \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i}, \quad t = 1, \dots, T.$$

where $S_t = 1$, if $\epsilon_t < 0$, and $S_t = 0$, if $\epsilon_t \geq 0$. Here T is the number of terms in the sequence, y_t denotes the endogenous variables, x_t the exogenous variables, b_o the mean, b the regression coefficients, ϵ_t the residuals, γ is the asymmetry parameter, h_t is the conditional variance, and ψ_t the information set of all information up to time t .

The routine nag_estimate_garchGJR provides an estimate for $\hat{\theta}$, the $(p + q + k + 3) \times 1$ parameter vector $\theta = (b_o, b^T, \omega^T)$ where $\omega^T = (\alpha_0, \alpha_1, \dots, \alpha_q, \beta_1, \dots, \beta_p, \gamma)$ and $b^T = (b_1, \dots, b_k)$.

mn, **nreg** (see Section 4) can be used to simplify the GARCH(p, q) expression in equation (1) as follows:

No Regression or Mean

$y_t = \epsilon_t$,
mn = 0,
nreg = 0, and
 θ is a $(p + q + 2) \times 1$ vector.

No Regression

$y_t = b_o + \epsilon_t$,
mn = 1,
nreg = 0, and
 θ is a $(p + q + 3) \times 1$ vector.

Note: if the $y_t = \mu + \epsilon_t$, where μ is known (not to be estimated by nag_estimate_garchGJR) then equation (1) can be written as $y_t^\mu = \epsilon_t$, where $y_t^\mu = y_t - \mu$. This corresponds to the case **No Regression or Mean**, with y_t replaced by $y_t - \mu$.

No Mean

$$y_t = x_t^T b + \epsilon_t,$$

$$\mathbf{mn} = 0,$$

$$\mathbf{nreg} = k \text{ and}$$

θ is a $(p + q + k + 2) \times 1$ vector.

4 Parameters

Note: for convenience **npar** will be used here to denote the expression $2 + \mathbf{q} + \mathbf{p} + \mathbf{mn} + \mathbf{nreg}$ representing the number of model parameters.

- 1: **yt[num]** – const double *Input*
 On entry: the sequence of observations, y_t , $t = 1, \dots, T$.

- 2: **x[num][tdx]** – const double *Input*
 On entry: row t of **x** contains the time dependent exogenous vector x_t , where $x_t^T = (x_t^1, \dots, x_t^k)$, for $t = 1, \dots, T$.

- 3: **tdx** – Integer *Input*
 On entry: the second dimension of the array **x** as declared in the function from which `nag_estimate_garchGJR` is called.
 Constraint: **tdx** \geq **nreg**.

- 4: **num** – Integer *Input*
 On entry: the number of terms in the sequence, T .
 Constraint: **num** \geq **npar**.

- 5: **p** – Integer *Input*
 On entry: the GARCH(p, q) parameter p .
 Constraint: **p** \geq 0.

- 6: **q** – Integer *Input*
 On entry: the GARCH(p, q) parameter q .
 Constraint: **q** \geq 1.

- 7: **nreg** – Integer *Input*
 On entry: the number of regression coefficients, k .
 Constraint: **nreg** \geq 0.

- 8: **mn** – Integer *Input*
 On entry: if **mn** = 1 then the mean term b_0 will be included in the model.
 Constraint: **mn** = 0 or **mn** = 1.

- 9: **theta[npar]** – double *Input/Output*
 On entry: the initial parameter estimates for the vector θ . The first element contains the coefficient α_0 , the next **q** elements contain the coefficients α_i , $i = 1, \dots, q$. The next **p** elements are the coefficients β_j , $j = 1, \dots, p$. The next element contains the asymmetry parameter γ . If **est_opt** = **Nag_Garch_Est_Initial_False** then (when **mn** = 1) the next term contains an initial estimate of the mean term b_0 and the remaining **nreg** elements are taken as initial estimates of the linear regression coefficients b_i , $i = 1, \dots, k$.

- On exit:* the estimated values $\hat{\theta}$ for the vector θ . The first element contains the coefficient α_o , the next **q** elements contain the coefficients α_i , $i = 1, \dots, q$. The next **p** elements are the coefficients β_j , $j = 1, \dots, p$. The next element contains the estimate for the asymmetry parameter γ . If **mn** = 1 then the next element contains an estimate for the mean term b_o . The final **nreg** elements are the estimated linear regression coefficients b_i , $i = 1, \dots, k$.
- 10: **se[npar]** – double *Output*
- On exit:* the standard errors for $\hat{\theta}$. The first element contains the standard error for α_o , the next **q** elements contain the standard errors for α_i , $i = 1, \dots, q$, the next **p** elements are the standard errors for β_j , $j = 1, \dots, p$. The next element contains the standard error for γ . If **mn** = 1 then the next element contains the standard error for b_o . The final **nreg** elements are the standard errors for b_j , $j = 1, \dots, k$.
- 11: **sc[npar]** – double *Output*
- On exit:* the scores for $\hat{\theta}$. The first element contains the score for α_o , the next **q** elements contain the score for α_i , $i = 1, \dots, q$, the next **p** elements are the scores for β_j , $j = 1, \dots, p$. The next element contains the score for γ . If **mn** = 1 then the next element contains the score for b_o . The final **nreg** elements are the scores for b_j , $j = 1, \dots, k$.
- 12: **covar[npar][tdc]** – double *Output*
- On exit:* the covariance matrix of the parameter estimates $\hat{\theta}$, that is the inverse of the Fisher Information Matrix.
- 13: **tdc** – Integer *Input*
- On entry:* the second dimension of the array **covar** as declared in the function from which `nag_estimate_garchGJR` is called.
- Constraint:* **tdc** \geq **npar**.
- 14: **hp** – double * *Input/Output*
- On entry:* If **est_opt** = **Nag_Garch_Est_Initial_False** then **hp** is the value to be used for the pre-observed conditional variance. If **est_opt** = **Nag_Garch_Est_Initial_True** then **hp** is not referenced.
- On exit:* If **est_opt** = **Nag_Garch_Est_Initial_True** then **hp** is the estimated value of the pre-observed conditional variance.
- 15: **et[num]** – double *Output*
- On exit:* the estimated residuals, ϵ_t , $t = 1, \dots, T$.
- 16: **ht[num]** – double *Output*
- On exit:* the estimated conditional variances, h_t , $t = 1, \dots, T$.
- 17: **lgf** – double * *Output*
- On exit:* the value of the log likelihood function at $\hat{\theta}$.
- 18: **stat_opt** – Nag_Garch_Stationary_Type *Input*
- On entry:* If **stat_opt** = **Nag_Garch_Stationary_True** then Stationary conditions are enforced. If **stat_opt** = **Nag_Garch_Stationary_False** then Stationary conditions are not enforced.
- 19: **est_opt** – Nag_Garch_Est_Initial_Type *Input*
- On entry:* If **est_opt** = **Nag_Garch_Est_Initial_True** then the routine provides initial parameter estimates of the regression terms (b_o, b^T). If **est_opt** = **Nag_Garch_Est_Initial_False** then the initial estimates of the regression parameters (b_o, b^T) must be supplied by the user.

- 20: **max_iter** – Integer *Input*
On entry: the maximum number of iterations to be used by the optimisation routine when estimating the GARCH(p, q) parameters. If **max_iter** is set to 0 then the standard errors, score vector and variance-covariance are calculated for the input value of θ in **theta**; however the value of θ is not updated.
Constraint: **max_iter** \geq 0.
- 21: **tol** – double *Input*
On entry: the tolerance to be used by the optimisation routine when estimating the GARCH(p, q) parameters.
- 22: **fail** – NagError * *Input/Output*
 The NAG error parameter (see the Essential Introduction).

5 Error Indicators and Warnings

NE_BAD_PARAM

On entry, parameter **stat_opt** had an illegal value.
 On entry, parameter **est_opt** had an illegal value.

NE_INT_ARG_LT

On entry, **nreg** must not be less than 0: **nreg** = $\langle value \rangle$.
 On entry, **q** must not be less than 1: **q** = $\langle value \rangle$.
 On entry, **p** must not be less than 0: **p** = $\langle value \rangle$.
 On entry, **max_iter** must not be less than 0: **max_iter** = $\langle value \rangle$.

NE_2_INT_ARG_LT

On entry, **tdx** = $\langle value \rangle$ while **nreg** = $\langle value \rangle$.
 These parameters must satisfy **tdx** \geq **nreg**.
 On entry, **tdc** = $\langle value \rangle$ while $2 + \mathbf{q} + \mathbf{p} + \mathbf{mn} + \mathbf{nreg} = \langle value \rangle$.
 These parameters must satisfy **tdc** $\geq 2 + \mathbf{q} + \mathbf{p} + \mathbf{mn} + \mathbf{nreg}$.
 On entry, **num** = $\langle value \rangle$ while $2 + \mathbf{q} + \mathbf{p} + \mathbf{mn} + \mathbf{nreg} = \langle value \rangle$.
 These parameters must satisfy **num** $\geq 2 + \mathbf{q} + \mathbf{p} + \mathbf{mn} + \mathbf{nreg}$.

NE_INVALID_INT_RANGE_2

Value $\langle value \rangle$ given to **mn** is not valid. Correct range is 0 to 1.

NE_MAT_NOT_FULL_RANK

Matrix X does not give a model of full rank.

NE_MAT_NOT_POS_DEF

Attempt to invert the second derivative matrix needed in the calculation of the covariance matrix of the parameter estimates has failed. The matrix is not positive-definite, possibly due to rounding errors.

NE_ALLOC_FAIL

Memory allocation failed.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments

6.1 Accuracy

Not applicable.

6.2 References

Engle R (1982) Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation *Econometrica* **50** 987–1008

Bollerslev T (1986) Generalised Autoregressive Conditional Heteroskedasticity *Journal of Econometrics* **31** 307–327

Engle R and Ng V (1993) Measuring and Testing the Impact of News on Volatility *Journal of Finance* **48** 1749–1777

Hamilton J (1994) *Time Series Analysis* Princeton University Press

7 See Also

None.

8 Example

This example program illustrates the use of `nag_estimate_garchGJR` to model a GARCH(1,1) sequence generated by `nag_generate_garchGJR` (`g05hmc`), a six step forecast is then calculated using `nag_forecast_garchGJR` (`g13ffc`).

8.1 Program Text

```

/* nag_estimate_garchGJR (g13fec) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 */
#include <nag.h>
#include <nag_stdlib.h>
#include <stdio.h>
#include <ctype.h>
#include <math.h>
#include <nagg05.h>
#include <nagg13.h>

int main(void)
{
    double *bx=0, *covar=0, *etm=0, facl, gamma, hp, *ht=0, *htm=0, lgf;
    double *param=0, *rvec=0, *sc=0, *se=0, *theta=0, tol;
    double mean, *x=0, xterm, *cvar=0, *yt=0;
    Integer exit_status = 0;
    Integer i, ip, iq, j, k, nt;
    Integer tdx, tdc, maxit, mn, num, num_startup, npar;
    Integer nreg, seed;
    Nag_Garch_Est_Initial_Type est_opt;
    Nag_Garch_Stationary_Type stat_opt;

```

```

Nag_Garch_Fcall_Type fcall;
NagError fail;

INIT_FAIL(fail);
num = 1000;
mn = 1;
mean = 4.0;
nreg = 2;
ip = 1;
iq = 1;
npar = iq + ip + 1;
nt = 6;
tdx = nreg;
tdc = npar+mn+nreg+1;

#define YT(I) yt[(I)-1]
#define THETA(I) theta[(I)-1]
#define SE(I) se[(I)-1]
#define SC(I) sc[(I)-1]
#define RVEC(I) rvec[(I)-1]
#define PARAM(I) param[(I)-1]
#define HTM(I) htm[(I)-1]
#define HT(I) ht[(I)-1]
#define ETM(I) etm[(I)-1]
#define BX(I) bx[(I)-1]
#define CVAR(I) cvar[(I)-1]
#define X(I,J) x[((I)-1) * tdx + ((J)-1)]
#define COVAR(I,J) covar[((I)-1) * tdc + ((J)-1)]

Vprintf ("g13fec Example Program Results \n\n");

if (!(bx = NAG_ALLOC (nreg, double))
    || !(covar = NAG_ALLOC ((npar+mn+nreg+1) * (npar+mn+nreg+1), double))
    || !(etm = NAG_ALLOC (num, double))
    || !(ht = NAG_ALLOC (num, double))
    || !(htm = NAG_ALLOC (num, double))
    || !(param = NAG_ALLOC (npar+mn+nreg+1, double))
    || !(rvec = NAG_ALLOC (40, double))
    || !(sc = NAG_ALLOC (npar+mn+nreg+1, double))
    || !(se = NAG_ALLOC (npar+mn+nreg+1, double))
    || !(theta = NAG_ALLOC (npar+mn+nreg+1, double))
    || !(cvar = NAG_ALLOC (nt, double))
    || !(x = NAG_ALLOC (num*nreg, double))
    || !(yt = NAG_ALLOC (num, double)))
{
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

seed = 11;
gamma = 0.1;
BX (1) = 1.5;
BX (2) = 2.5;

for (i = 1; i <= num; ++i)
{
    fac1 = (double) i *0.01;

```

```

        X (i, 2) = sin (fac1) * 0.7 + 0.01;
        X (i, 1) = fac1 * 0.1 + 0.5;
    }

PARAM (1) = 0.4;
PARAM (2) = 0.1;
PARAM (3) = 0.7;

fcall = Nag_Garch_Fcall_True;
g05cbc(seed);
num_startup = 200;
g05hmc (num_startup, ip, iq, &PARAM (1), gamma, &HT (1), &YT (1),
        fcall, &RVEC (1), &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from g05hmc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
fcall = Nag_Garch_Fcall_False;
g05hmc (num, ip, iq, &PARAM (1), gamma, &HT (1), &YT (1),
        fcall, &RVEC (1), &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from g05hmc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

for (i = 1; i <= num; ++i)
{
    xterm = 0.0;
    for (k = 1; k <= nreg; ++k)
xterm += X (i, k) * BX (k);

    if (mn == 1)
YT (i) = mean + xterm + YT (i);
    else
YT (i) = xterm + YT (i);
}

est_opt = Nag_Garch_Est_Initial_True;
stat_opt = Nag_Garch_Stationary_True;
maxit = 50;
tol = 1e-12;

for (i = 1; i <= npar; ++i)
    THETA (i) = PARAM (i) * 0.5;
THETA (npar + 1) = gamma * 0.5;
if (mn == 1)
    THETA (npar + 2) = mean * 0.5;

for (i = 1; i <= nreg; ++i)
    THETA (npar + 1 + mn + i) = BX (i) * 0.5;

g13fec (&YT (1), &X (1, 1), tdx, num, ip, iq, nreg, mn,
        &THETA (1), &SE (1), &SC (1), &COVAR (1, 1), tdc, &hp,
        &ETM (1), &HTM (1), &lgf, stat_opt, est_opt, maxit, tol, &fail);
if (fail.code != NE_NOERROR)

```

```

    {
        Vprintf("Error from g13fec.\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    Vprintf ("          Parameter estimates          Standard errors          Correct va-
lues\n");
    for (j = 1; j <= npar; ++j)
        Vprintf ("%20.4f                (%6.4f) %20.4f\n", THETA (j), SE (j),
PARAM(j));

    Vprintf ("%20.4f                (%6.4f) %20.4f\n", THETA (npar+1), SE (npar+1),
gamma);
    if (mn)
        Vprintf ("%20.4f                (%6.4f) %20.4f\n", THETA (npar+2), SE (npar+2),
mean);
    for (j = 1; j <= nreg; ++j)
        Vprintf ("%20.4f                (%6.4f) %20.4f\n", THETA (npar+1+mn+j), SE(n-
par+1+mn+j), BX(j));

/* now forecast nt steps ahead */

gamma = THETA(npar+1);

g13ffc(num,nt,ip,iq,&THETA(1),gamma,&CVAR(1),&HTM(1),&ETM(1),&fail);

Vprintf ("\n%ld step forecast = %8.4f\n",nt, CVAR(nt));

END:
if (bx) NAG_FREE (bx);
if (covar) NAG_FREE (covar);
if (etm) NAG_FREE (etm);
if (ht) NAG_FREE (ht);
if (htm) NAG_FREE (htm);
if (param) NAG_FREE (param);
if (sc) NAG_FREE (sc);
if (se) NAG_FREE (se);
if (theta) NAG_FREE (theta);
if (cvar) NAG_FREE (cvar);
if (x) NAG_FREE (x);
if (yt) NAG_FREE (yt);
if (rvec) NAG_FREE (rvec);

return exit_status;

}

```

8.2 Program Data

None.

8.3 Program Results

g13fec Example Program Results

| Parameter estimates | Standard errors | Correct values |
|---------------------|-----------------|----------------|
| 0.4326 | (0.1356) | 0.4000 |
| 0.0685 | (0.0333) | 0.1000 |
| 0.7173 | (0.0672) | 0.7000 |
| 0.1326 | (0.0553) | 0.1000 |
| 4.1205 | (0.1730) | 4.0000 |
| 1.3950 | (0.1658) | 1.5000 |
| 2.4518 | (0.1037) | 2.5000 |

6 step forecast = 2.2549
